Last Name:

First Name:

SCIPER:

Practice Problems: Exercise 6 – Microengineering 110

Spring 2025

Prof. Vivek Subramanian

1. A gas station has both self-service and full-service islands. In the self-service island, customers fill gas themselves.   In the full-service island, an attendant fills the gas On each island, there is a single regular petrol pump with two hoses. Let X denote the number of hoses being used on the self-service island at a particular time, and let Y denote the number of hoses on the full-service island in use at that time. The joint pmf of X and Y appears in the accompanying tabulation.

		\(y\)		
$p(x, y)$		0	1	2
	0	.10	.04	.02
x	1	.08	.20	.06
	2	.06	.14	.30

   a.   What is P(X = 1 and Y = 1)?

   b.   Compute P(X <= 1 and Y <= 1).

   c.   Compute the probability of X ≠ 0 and Y ≠ = 0.

   d.   Compute the marginal pmf of X and of Y. Using $p_X(x)$, what is P(X <= 1)?

   e.   e. Are X and Y independent rv's?

2. Two components of a computer have the following joint pdf for their useful lifetimes X and Y:

$$f(x, y) = \begin{cases} xe^{-x(1+y)} & x \geq 0 \text{ and } y \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

a. What is the probability that the lifetime X of the first component exceeds 3?

b. What are the marginal pdf's of X and Y? Are the two lifetimes independent?

c. What is the probability that the lifetime of at least one component exceeds 3?

3. An instructor has given a short quiz consisting of two parts. For a randomly selected student, let X = the number of points earned on the first part and Y = the number of points earned on the second part. Suppose that the joint pmf of X and Y is given in the accompanying table.

$p(x, y)$		0	5	10	15
	0	.02	.06	.02	.10
x	5	.04	.15	.20	.10
	10	.01	.15	.14	.01

(column header row labeled y over 0, 5, 10, 15)

a. If the score recorded in the grade book is the total number of points earned on the two parts, what is the expected recorded score E(X + Y)?

b. If the maximum of the two scores is recorded, what is the expected recorded score?

c. Compute the covariance for X and Y